

# Bounds on Kaluza-Klein excitations of the SM vector bosons from electroweak tests

Alessandro Strumia

*Dipartimento di fisica, Università di Pisa and INFN, sezione di Pisa, I-56126 Pisa, Italia*

## Abstract

Within a minimal extension of the SM in  $4 + 1$  dimensions, we study how Kaluza Klein excitations of the SM gauge bosons affect the electroweak precision observables. Asymmetries in  $Z$  decays provide the dominant bound on the compactification scale  $M$  of the extra dimension. If the higgs is so light that will be discovered at LEP2, we find the following 95% C.L. bounds:  $M > 3.5$  TeV (if the higgs lives in the extra dimension) and  $M > 4.3$  TeV (if the higgs is confined to our 4 dimensions). In the second case, Kaluza Klein modes give ‘universal’ corrections and a good fit of precision data can be obtained with a heavier higgs (up to 500 GeV) and with a smaller  $M > 3.4$  TeV.

## 1 Introduction

It has been recently realized that no known experimental constraint excludes extra spatial dimensions so large that can be discovered at future experiments, and that no known theoretical constraint excludes that this possibility be realized within string theory, with a string scale in the TeV range [1]. A scenario with such a low string scale can be motivated as a new possible solution of the naturalness problem of having a higgs much lighter than the Planck scale.

String excitations presumably generate a set of non renormalizable operators (NRO) suppressed by powers of the string scale. Even assuming that only operators that conserve baryon number, lepton number, hadronic and leptonic flavour and CP are present, dimension 6 operators that affect the electroweak precision observables (EWPO) must be suppressed by a factor  $1/\Lambda^2$  around  $1/v^2 N_Z^{1/2}$  (where  $N_Z \sim 10^7$  is the number of observed  $Z$  decays and  $v = 175$  GeV). A computation of all relevant operators [2] indeed shows that few of these operators (and probably a generic set of them) must be suppressed by  $\Lambda \approx 10$  TeV. We cannot however derive interesting implications from such bounds: are they sufficiently strong to forbid observable Kaluza-Klein (KK) graviton effects [3] at LHC? Is a higgs with  $v = 175$  GeV natural if  $\Lambda \gtrsim 10$  TeV? Similarly we cannot establish if observations about the 1987 supernova [4], flavour

violation, CP violation, neutrino masses, proton decay, nucleosynthesis and cosmic baryon asymmetry are compatible with a so light string scale. Unfortunately string theory is currently an example of a theory with no parameters that makes no calculable predictions.

In the following we will forget the possible but currently uncontrollable NRO of string origin and we will study the bounds set by EWPO on the scale of possible new extra dimensions where SM gauge bosons propagate. This kind of extra dimensions are interesting because, if larger than the string scale, modify the string predictions for the gauge couplings in a way that qualitatively resembles the observed values [5]. Beyond affecting the parameters of the SM, KK excitations of the gauge bosons also give minimal computable corrections to EWPO [6, 7, 8].

In section 2 we briefly recall a concrete minimal extension of the SM to 5 dimensions [9] (‘M5SM’) and we write the effective Lagrangian below the compactification scale  $M$  in terms of the complete set of non renormalizable operators that affect EWPO used in [2] and listed in the appendix. Since we do not know if the higgs field should be confined to our 4 dimensions or can propagate in the extra dimensions, the model contains two higgs doublets with the two different behaviors. Only one unknown parameter is necessary to take into account the resulting uncertainty in low energy effects [7].

$M_Z$	$= 91.187 \text{ GeV}$
$G_\mu$	$= 1.1664 \cdot 10^{-5} \text{ GeV}^{-2}$
$\Gamma_Z$	$= (2.4939 \pm 0.0024) \text{ GeV}$
$R_h$	$= 20.765 \pm 0.026$
$R_b$	$= 0.21680 \pm 0.00073$
$\sigma_h$	$= (41.491 \pm 0.058) \text{ nb}$
$s_{\text{eff}}^2$	$= 0.23157 \pm 0.00018$
$M_W$	$= (80.394 \pm 0.042) \text{ GeV}$
$m_t$	$= (174.3 \pm 5.1) \text{ GeV}$
$\alpha_3(M_Z)$	$= 0.119 \pm 0.004$
$\alpha_{\text{em}}^{-1}(M_Z)$	$= 128.92 \pm 0.036$

Table 1: *Electroweak precision observables.*

In section 3 we derive bounds on  $M$  from a global fit of the most recent data about electroweak precision observables [10, 11], listed in table 1. The list of observables includes the Fermi constant measured in  $\mu$  decay and the  $Z$  mass (known with great precision), the  $W$  and top masses, the various  $Z$  widths, the various asymmetries in  $Z$  decays grouped into an ‘effective  $s_W$ ’ and the values of the electromagnetic and strong gauge couplings. The list does not include atomic parity violation (APV), the neutrino-nucleon cross sections and tests of quark-lepton universality. Their inclusion would not shift our final results (the best-fit regions in the  $m_h$  and  $M$  plane shown in fig. 1) in a significant way; however, since there is now some discrepancy between the measured value of APV [12, 13] and the SM prediction, the inclusion of APV would strongly deteriorate the quality of the SM and M5SM fits [14]\*.

## 2 The model

In this section we briefly recall a concrete minimal extension of the SM to 5 dimensions [9] and we com-

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\*Recent measurements about the Cesium atomic structure [12] correct previous data and allow to reduce the atomic structure uncertainties, that still remain the largest uncertainty in the SM prediction for APV in Cesium. The measured value of APV [13],  $Q_W = -72.06 \pm 0.28_{\text{exp}} \pm 0.34_{\text{th}}$ , is now significantly smaller in modulus (2.5 ‘standard deviations’, if one adds experimental and theoretical errors in quadrature) than the SM prediction. Not including APV in the fit we derive more safe bounds on the mass of KK modes (that increase the amount of APV). Its inclusion would strongly increase the minimal value of the  $\chi^2$ , both in the SM and in the M5SM, and would allow to derive strong bounds on these models [14] based on the ‘goodness of the fit’. It would instead not shift in a significant way the ‘confidence intervals’ on  $m_h$  and  $M$  that we study in this paper: the 0.6% error on APV is still significantly larger than the  $\sim 0.1\%$  error on various cleaner electroweak observables listed in table 1. For example, in a pure SM fit, the inclusion of APV reduces by only 8 GeV (i.e. by  $\sim 7\%$ ) the best fit value of  $m_h$  while increases by  $\sim 250\%$  the value of  $(\min \chi^2)/\text{d.o.f.}$ . ‘Confidence intervals’ are commonly employed to report experimental data, in place of intervals based on ‘goodness of the fit’, due to their stability with respect to rare statistical fluctuations and/or underestimated systematic errors.

pute how the KK excitations of the SM gauge bosons affect the EWPO. The model contains one extra dimension compactified on  $\mathcal{S}_1/\mathcal{Z}_2$  where the SM gauge fields can propagate (the circle  $\mathcal{S}_1$  has radius  $R = 1/M$ ; the  $\mathcal{Z}_2$  symmetry ensures that the massless spectrum only contains the SM fields). The SM fermions are instead confined to 4 dimensions. The higgs doublet could follow both possibilities. Since we do not know which possibility (if any) is the physical one, the model has two higgs doublets  $H_4$  and  $H_5$ :  $H_4$  is confined to our 4 dimensions while  $H_5$  can propagate into the extra dimension. Both higgs doublets could contribute to EWSB. Their effects can be parameterized in terms of an angle  $\beta$  [7]

$$\langle H_4 \rangle = (0, v \sin \beta), \quad \langle H_5 \rangle = (0, v \cos \beta)$$

where  $v = 175 \text{ GeV}$  and  $\beta$  has nothing to do with the  $\beta$  used in supersymmetric models.

At tree level the KK excitations  $A_\mu^n$  of the SM gauge fields, with mass  $M_n = nM$  ( $n = 1, \dots, \infty$ ), couple to SM particles with the Lagrangian interaction  $\sqrt{2}A_\mu^n J_\mu$  where  $J_\mu$  are the contributions from four dimensional fields to the usual gauge currents. Conservation of momentum in the extra dimension forbids the five-dimensional fields to appear in the currents (see [6, 7] for more details). All our analysis of precision data could be rephrased in terms of excited gauge bosons  $A_\mu^*$  with couplings  $g^* = \sqrt{2}g$ . Since KK modes are currently more popular than composite particles, we will perform our analysis with the normalization factors appropriate for KK modes.

The fact that KK modes couple to observed particles in a way similar to the SM vector bosons has been used to compute the KK corrections to various EWPO [6, 7, 8]. Here we follow a less direct strategy because we prefer to use the results in [2], where the effects of a complete set of 10 non renormalizable operators (recalled in the appendix) on EWPO have been listed.

Thus we need to write the effective Lagrangian for the SM fields obtained integrating out the KK excitations. The first KK level gives

$$\mathcal{L}_1 = -\frac{1}{M^2}(J_\mu^a J_\mu^a + J_\mu^B J_\mu^B + J_\mu^G J_\mu^G) \quad (1)$$

In this model the  $n$ -th KK mode gives (at tree level)  $\mathcal{L}_n = \mathcal{L}_1/n^2$ , so that summing over  $n$  one obtains  $\mathcal{L}_{\text{eff}} = \frac{\pi^2}{6}\mathcal{L}_1$ . With one extra spatial dimension the first few terms dominate. With more than one extra dimension the sum over KK excitations is divergent. In both cases the string scale cannot be much larger than the compactification scale and will cut-off the sum at  $n \lesssim M_{\text{string}}/M$ . In general we do not know the numerical factor that relates  $\mathcal{L}_{\text{eff}}$  to  $\mathcal{L}_1$ . In order to avoid a normalization of  $M$  different from previous analysis [7], we keep the factor  $\pi^2/6$ . This

model dependent normalization factor is not much relevant (but not completely irrelevant) when comparing LEP1 bounds with capabilities of LHC.

The gluonic current does not affect electroweak precision observables so that we neglect it in the following. The currents coupled to the KK modes of the  $SU(2)_L$  and  $U(1)_Y$  gauge vector bosons are

$$J_\mu^a = \frac{g_2}{2} \left[ \sum_{L,Q} (\bar{F} \gamma_\mu \tau^a F) + (i H_4^\dagger \tau^a D_\mu H_4 + \text{h.c.}) \right]$$

$$J_\mu^B = g_Y \left[ \sum (Y_F \bar{F} \gamma_\mu F) + Y_H (i H_4^\dagger D_\mu H_4 + \text{h.c.}) \right]$$

The sum in  $J^a$  runs over the fermionic doublets  $F = L, Q$ , while the sum in  $J^B$  runs over all the SM fermions. In the standard notation that we employ the hypercharges  $Y_F$  are

$$\{Y_L, Y_Q, Y_E, Y_U, Y_D\} = \left\{-\frac{1}{2}, \frac{1}{6}, -1, \frac{2}{3}, -\frac{1}{3}\right\}.$$

The effective Lagrangian can thus be written in terms of the operators in the appendix as

$$\mathcal{L}_1 = -\frac{g_2^2}{2M^2} \left[ -\mathcal{O}_{LL} + \sin^2 \beta (\mathcal{O}'_{HL} + \mathcal{O}'_{HQ}) \right] +$$

$$-\frac{g_Y^2}{M^2} \sin^2 \beta \left[ \sum Y_F \mathcal{O}_{HF} + \mathcal{O}_H \sin^2 \beta \right] \quad (2)$$

up to operators that do not affect the electroweak precision observables that we consider. Using the results in [2] it is straightforward to compute the corrections from this set of NRO to the various EWPO.

Two limiting cases are of particular interest. If  $\sin \beta = 0$  (EWSB entirely due to a five-dimensional Higgs  $H_5$ ) the relevant effective Lagrangian contains a single operator

$$\mathcal{L}_1 = \frac{g_2^2}{2M^2} \mathcal{O}_{LL}$$

again up to terms that do not affect EWPO so that the bounds on it are the same as those in [2]. The limiting case  $\sin \beta = 0$  is however problematic since it seems unlikely that the Yukawa couplings of a 5 dimensional Higgs to the top quark, suppressed by a factor  $\sim R^{1/2}$ , can generate the top mass.

More interesting is the opposite limit,  $\sin \beta = 1$  (EWSB entirely due to a four-dimensional Higgs  $H_4$ ;  $H_5$  is either absent or irrelevant). In this case the effective Lagrangian can be rewritten in the simple form

$$\mathcal{L}_1 = -\frac{1}{M^2} (\mathcal{O}_{WW} + \mathcal{O}_{BB}) \quad (3)$$

once again up to terms that do not affect EWPO<sup>†</sup>

<sup>†</sup>We explicitly demonstrate this fact in the abelian case. Applying the Bianchi identities  $\partial_\alpha B_{\mu\nu} = -\partial_\mu B_{\nu\alpha} - \partial_\nu B_{\alpha\mu}$  to one of the two factors in  $\mathcal{O}_{BB} = \frac{1}{2}(\partial_\alpha B_{\mu\nu})(\partial_\alpha B_{\mu\nu})$  and integrating by parts, one obtains products of the combinations  $\partial_\alpha B_{\alpha\nu}$  that appear in the classical equation of motion of the  $B_{\alpha\nu}$  gauge field. In this way one obtains  $\int d^4x \mathcal{O}_{BB} = \int d^4x J_\mu^B J_\mu^B$  up to surface terms.

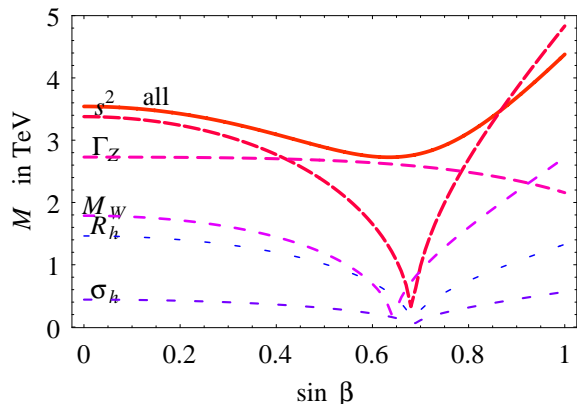


Figure 2: 95% C.L. lower bounds on the compactification scale  $M$  from electroweak data for  $m_h = 100$  GeV as function of  $\sin \beta$  ( $\sin \beta = 1$  corresponds to a higgs confined to 4 dimensions). The continuous line shows the bound from the global fit, while dashed lines show the bounds from single measurements.

Beyond proving a check of the computation, eq. (3) will be useful for interpreting the bounds that we will find in the limiting case  $\sin \beta = 1$ . An interesting analysis of these two operators can be found in [15].

It would be simple to extend the analysis to the more general models considered in [16]: for example if also the leptons can propagate in the extra dimensions one should omit all operators involving leptons from the effective Lagrangian (2); if instead the  $SU(2)_L$  gauge bosons are confined to our 4 dimensions one should omit their  $J_\mu^a J_\mu^a$  contribution.

### 3 Results

We begin our analysis making the simplifying assumption that the higgs is so light that will be observed at LEP2 or Tevatron in the next years. For fixed  $m_h \approx 100$  GeV, we can compute a  $\chi^2(M, \sin \beta)$  by minimizing the full  $\chi^2$  with respect to  $m_t$ ,  $\alpha_{\text{strong}}$  and  $\alpha_{\text{em}}$  for each value of  $M$  and  $\sin \beta$ .

In fig. 2 we show the resulting  $2\sigma \approx 95\%$  C.L. bound (i.e.  $\Delta\chi^2 = 3.85$ ) on the KK mass  $M$  as function of  $\sin \beta$  (continuous line). The dashed lines show the bounds from the single experimental data that we have fitted (omitting the less relevant ones). The apparent difference with respect to an analogous fig. in [7] is only due to a different choice of a minimal set of experimental data (with our choice there are no significant correlations between the data). The strongest bound comes from the ‘effective  $s_W$ ’ extracted at LEP and SLD from various asymmetries in  $Z$  decays. Due to the accidentally small SM value of the vector coupling of leptons to the  $Z$ , these asymmetries are particularly sensible to new physics.

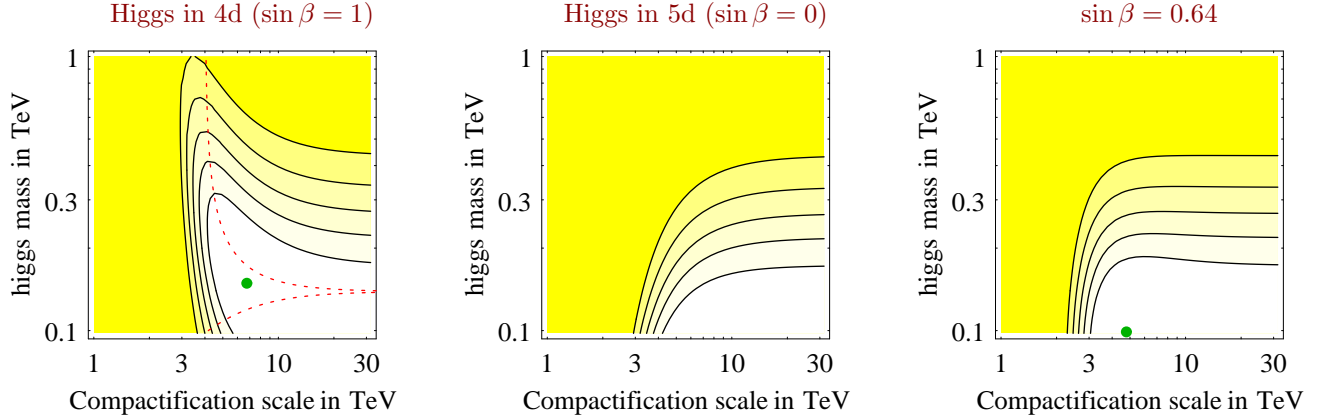


Figure 1: *Bounds on the compactification scale  $M$  and on the higgs mass in three extensions of the SM to five dimensions: (a) with a higgs doublet confined to 4 dimensions; (b) with a higgs doublet that can propagate to the extra dimension; (c) a combination of (a) and (b). The level curves correspond to  $\Delta\chi^2 = \{1, 2.3, 3.8, 6, 9.2\}$ .*

More precisely KK modes affect the correlation between this ‘effective  $s_W^2$ ’ and the  $s_W^2$  derived from  $\alpha_{\text{em}}$ , the  $Z$  mass,  $M_Z$ , and the Fermi constant for  $\mu$  decay,  $G_\mu$ , as  $s_W^2 c_W^2 = \pi\alpha_{\text{em}}/\sqrt{2}G_\mu M_Z^2$ .

From fig. 2 we see that the 95% C.L. bound on the compactification scale  $M = 1/R$  is  $M > 3.5$  TeV in presence of one higgs doublet that propagates in the extra dimension, and  $M > 4.3$  TeV in the simplest case of EWSB entirely due to a higgs confined to four dimensions. Asymmetries in  $Z$  decays give the dominant bound, except when  $\sin^2 \beta$  is close to  $\sqrt{2} - 1$ , where some cancellations take place. In this case the measurement of the  $Z$  width provides the weaker bound  $M > 2.7$  TeV.

It is interesting to perform a more complicated analysis and study the EWPO bound on  $M$  in association with the EWPO bound on the higgs mass. Since EWPO imply a light higgs only in absence of new physics, both the upper bound on  $m_h$  and the lower bound on  $M$  could be somewhat relaxed. Few cases where this happens can be found in [17, 18, 2]. This happens also in the case that we are studying if  $\sin \beta$  is large enough (i.e. in the simplest scenario with only a 4-dimensional higgs). In this case KK corrections decrease the predicted value of the ‘effective  $s_W^2$ ’ that gives the strongest bound on  $M$ , while a heavy higgs increases it. The measurement of the effective  $s_W^2$  thus allows to have light KK modes and a heavy higgs, with an appropriate cancellation of their effects. Since other significant observables, like the  $Z$  width, do not allow such cancellation, only a limited weakening of the limits on  $M$  and  $m_h$  is possible. The contour plots of the  $\Delta\chi^2(M, m_h)$  in fig.s 1 show the allowed regions in the  $(M, m_h)$  plane for different values of  $\sin \beta$ . Fig. 1a refers to a higgs confined to four dimensions; while fig. 1b is valid in the opposite (problematic) limit where only a 5 dimensional higgs exists. No cancellations between heavy higgs

and KK effects happen in this second case. Finally fig. 1c refers to the case  $\sin^2 \beta = \sqrt{2} - 1 = 0.64$  that has the weakest bound in fig. 2 due to some cancellations between KK effects.

The  $\Delta\chi^2(M, m_h)$  plotted in fig.s 1 is precisely defined as follows. For a given value of  $\sin \beta$ , and for each value of  $M$  and  $m_h$  we compute a  $\chi^2(M, m_h)$  minimizing the full  $\chi^2$  with respect to  $m_t$ ,  $\alpha_{\text{strong}}$  and  $\alpha_{\text{em}}$ .  $\Delta\chi^2$  is defined as  $\chi^2(M, m_h)$  minus its minimum value, whose location is marked in fig.s 1 with a disk. The chosen contour levels correspond to the conventional 68%, 95% and 99% ‘confidence levels’ on  $m_h$ , on  $M$  or on the couple  $(m_h, M)$ .<sup>‡</sup>

Fig. 1a also contains a dashed line. It has been plotted to illustrate how natural is having a higgs lighter than the compactification scale. At the right of the dashed line, the quadratically divergent one loop correction to the squared higgs mass, computed in the SM [19] and cutoffed at  $M$ , is more than ten times larger than the squared higgs mass itself (so that a fine tuning  $\gtrsim 10$  is required). We see that from the point of view of this naturalness problem, a heavy higgs is not more natural than a light one, due to its larger self-coupling (the particular behavior of the dashed line for  $m_h$  around 130 GeV is due to a cancellation between the various SM loop effects). Supersymmetry could still be necessary to justify the lightness of the higgs. No change in our analysis is necessary in the supersymmetric case, since experimental bounds on sparticles [11, 20] guarantee that the EWPOs more crucial for our analysis are not affected by significant supersymmetric loop effects [21]. Maybe supersymmetry could force the higgs to be light. This is however not guar-

<sup>‡</sup>These results can be easily translated into the results of a Bayesian analysis. The contour levels correspond to values of  $-2\ln p$ , where  $p$  is the Bayesian probability density (normalized to be one at the best fit point) in  $\ln m_h$  and  $1/M^2$  obtained assuming a flat prior distribution in the same variables.



anteed in presence of non renormalizable operators: the superpotential can contain a quartic term  $W_4 = (H_u H_d)^2 / 2\Lambda$ , and the Lagrangian can contain the corresponding ‘ $A$ -term’  $AW^{(4)}$ . Here  $H_u$  and  $H_d$  are the two higgs superfields required by supersymmetry. If  $\Lambda \sim (\text{few TeV})$  and the soft terms  $\mu, A$  are larger than  $v$ , the light higgs mass  $m_h$  could receive significant corrections. In this case it is easy to compute  $m_h$  analytically using the results (and the notations) of [22], where the general two-higgs-doublet potential is studied. The NRO terms induce extra contributions to the quartic couplings

$$\delta\lambda_6 = \delta\lambda_7 = \frac{\mu^*}{\Lambda}, \quad \delta\lambda_5 = -\frac{A}{\lambda}.$$

The light higgs is no longer constrained to be smaller than  $M_Z |\cos 2\beta|$  at tree level as in the renormalizable case:  $|\mu/\Lambda| \gtrsim 0.1$  is sufficient for having  $m_h \gtrsim 100 \text{ GeV}$  even at  $\tan\beta = 1$ .

## 4 Conclusion

Within a minimal extension of the SM to 5 dimensions we have studied the bounds on the size of the compactified extra dimension, due to minimal corrections to EWPO mediated by KK excitations of the SM vector bosons. If the higgs is so light that will be discovered at LEP2 (a possibility suggested by EWPO themselves and by supersymmetric models), we find the following 95% C.L. bounds on the radius  $R = 1/M$  of the extra dimension where gauge fields can propagate:  $M > 3.5 \text{ TeV}$  (if the higgs lives in the extra dimension)  $M > 4.3 \text{ TeV}$  (if the higgs is confined to our four dimensions).

In the second case the bound can be a bit relaxed because KK corrections allow a good fit of all precision data even in presence of a heavier higgs, up to  $\sim 500 \text{ GeV}$ . Accidental cancellations that compensate in many precision observables the loop corrections of a heavy higgs with the KK corrections are not very unlikely in a case, like this one, where both corrections affect the precision observables in an ‘universal’ way. This means that all the effects can be confined to the propagators of the gauge bosons (as in eq. (3)) so that all the experimental data are affected only through few parameters (usually called  $S, T, U$  [23] or  $\epsilon_1, \epsilon_2, \epsilon_3$  [24]). No compensation happens in  $\epsilon_1$  (that is reduced both by heavy higgs and KK corrections), but its experimental value is a bit lower than the best fit SM value.

In all cases (except when  $\sin\beta$  is close to 0.65) the strongest bound on  $M$  comes from asymmetries in  $Z$  decays, that mainly depend on the ‘effective  $s_W^2$ ’ that parameterizes the leptonic couplings of the  $Z$ . An improvement in its measure (or a shift in its central value) would thus affect our results. An higher central value (like the one measured at LEP) would give stronger constraints, while a lower value

(like the one measured at SLD) could even indicate the presence of a signal, if the error will be reduced by a factor  $2 \div 3$ . On the contrary no significant improvement of the bound on the compactification scale  $M$  will result from an improved measurement of the  $W$  mass: even with a  $\pm 15 \text{ MeV}$  error the measure of  $M_W$  will continue to give a subdominant bound. Atomic parity violation gives negligible bounds on extra dimensions, and receives negligible corrections.

Comparable bounds are present in more general models, for example in presence of a single extra dimension with the substructure proposed in [25] to suppress proton decay. With more than one extra dimension KK modes with mass close to the string scale give the dominant effect, so that the details of the string ‘model’ affect EWPO. The larger multiplicity of KK modes probably implies stronger bounds on the compactification radii.

These LEP bounds make extremely unlikely that KK effects can be observed at Tevatron. On the contrary LHC with high luminosity ( $100 \text{ fb}^{-1}$ ) can see KK effects up to  $M \sim (6 \div 7) \text{ TeV}$  [26]. If a signal will be found, it could be difficult to distinguish directly KK modes from a compositeness excitation of the  $W$ . For example the effects of the next KK levels ( $n > 1$ ) could be too small to be seen.

**Note added** The same kind of analysis presented in section 3 has been performed in a recent paper [27]. Our conclusions and our bounds on  $M$  agree with their results.

**Acknowledgments** We are grateful to Riccardo Barbieri and Riccardo Rattazzi for many useful discussions.

## A NRO operators

Here we briefly recall few technical details, discussed in a more complete way in [2], where notations are precisely defined.

The following 10 operators are a minimal set of gauge invariant, flavour symmetric, CP-even operators of dimension 6 that can affect the EWPO:

$$\begin{aligned} \mathcal{O}_{WB} &= (H^\dagger \tau^a H) W_{\mu\nu}^a B_{\mu\nu} \\ \mathcal{O}_H &= |H^\dagger D_\mu H|^2 \\ \mathcal{O}_{LL} &= \frac{1}{2} (\bar{L} \gamma_\mu \tau^a L)^2 \\ \mathcal{O}'_{HL} &= i(H^\dagger D_\mu \tau^a H) (\bar{L} \gamma_\mu \tau^a L) + \text{h.c.} \\ \mathcal{O}'_{HQ} &= i(H^\dagger D_\mu \tau^a H) (\bar{Q} \gamma_\mu \tau^a Q) + \text{h.c.} \\ \mathcal{O}_{HL} &= i(H^\dagger D_\mu H) (\bar{L} \gamma_\mu L) + \text{h.c.} \\ \mathcal{O}_{HQ} &= i(H^\dagger D_\mu H) (\bar{Q} \gamma_\mu Q) + \text{h.c.} \\ \mathcal{O}_{HE} &= i(H^\dagger D_\mu H) (\bar{E} \gamma_\mu E) + \text{h.c.} \\ \mathcal{O}_{HU} &= i(H^\dagger D_\mu H) (\bar{U} \gamma_\mu U) + \text{h.c.} \\ \mathcal{O}_{HD} &= i(H^\dagger D_\mu H) (\bar{D} \gamma_\mu D) + \text{h.c.} \end{aligned}$$

This set is minimal in the sense that any other operator that contributes to the EWPO of table 1 can be written as a combination of them, up to operators that give null contribution. For our purposes the most general effective Lagrangian with dimension six operators can thus be written as

$$\mathcal{L}_{\text{NRO}} = \sum_{i=1}^{10} c_i \mathcal{O}_i.$$

The coefficients  $c_i$  appropriate for our analysis are given in eq. (2). The contributions from the single operators to the form factors  $\delta e_i$ ,  $\delta G_{\text{VB}}$ ,  $\delta g_{Vf}$  and  $\delta g_{Af}$  (precisely defined, e.g., in [24]) are listed in table 2 of [2]. The form factors affect the EWPO in an obvious way; since the computation is however not immediate, the explicit expressions given in [2] could be useful. Fitting the  $\epsilon_i$  (or the  $S, T, U$ ) parameters would be much simpler; however in presence of ‘non universal’ corrections it is not a correct approximation and a more cumbersome fit of the the EWPO is necessary.

The two operators

$$\mathcal{O}_{WW} = \frac{1}{2}(D_\rho W_{\mu\nu}^a)^2, \quad \mathcal{O}_{BB} = \frac{1}{2}(\partial_\rho B_{\mu\nu})^2$$

can be written as a combination of the ten operators listed above, plus operators that do not affect EWPO. They are however interesting for our analysis (see eq.s (2) and (3)). If they are present in the effective Lagrangian with coefficients  $c_{WW}$  and  $c_{BB}$ , the form factors receive the following corrections

$$\begin{aligned} \delta e_2 &= c_{WW} g_2^2 v^2 \tan^2 \theta_W, \\ \delta e_4 &= +g_2^2 v^2 (c_{BB} + c_{WW} \tan^2 \theta_W), \\ \delta e_5 &= -g_2^2 v^2 (c_{WW} + c_{BB} \tan^2 \theta_W) \end{aligned}$$

Since few of these form factors were zero in [2], eq. (2) of [2] must be generalized as [21]

$$\begin{aligned} \delta \epsilon_1 &= \delta e_1 - \delta e_5 - \delta G_{\text{VB}} \\ \delta \epsilon_2 &= \delta e_2 - s_W^2 \delta e_4 - c_W^2 \delta e_5 - \delta G_{\text{VB}} \\ \delta \epsilon_3 &= \delta e_3 + c_W^2 \delta e_4 - c_W^2 \delta e_5. \end{aligned}$$

## References

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